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High-temperature plasmas in clusters of galaxies

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Abstract. Recent developments in the study of high-temperature plasmas in clusters of galaxies are reviewed. The Compton scattering of the permeating 2.7 K cosmic microwave background photons by high-temperature electrons of energy ∼10 keV produces a distortion in the Planck distribution of the 2.7 K cosmic microwave background photons. This effect, which has been termed the Sunyaev–Zel'dovich effect, has been studied extensively in recent years both theoretically and observationally. We will give a review of the study on this subject.

1. Introduction

The scope of the Oji International Seminar on the 'Quest for New Physical Phases under Extreme Conditions', to which this Special Issue is devoted, is very broad, although many of the papers presented at the Seminar deal with condensed matter physics in one way or another. In the present paper we wish to discuss a different aspect of the research in this field: we wish to discuss the interaction of very-low-temperature (2.7 K) photons with very-high-temperature (∼10 keV) electrons. This interesting encounter is realized in clusters of galaxies which contain about 100 galaxies and very hot (∼10 keV), tenuous ($\sim 10^{-3}$ cm^{-3}) plasmas [1].

The Compton scattering of the permeating 2.7 K cosmic microwave background photons by the high-temperature electrons of energy ∼10 keV produces a distortion in the Planck distribution of the 2.7 K cosmic microwave background photons. This effect, which has been termed the Sunyaev–Zel'dovich effect, has been studied extensively in recent years both theoretically and observationally.

In section 2 we will give a historical account of the Sunyaev–Zel'dovich effect. From section 3 onwards we will review the recent theoretical work on the relativistic corrections to the Sunyaev–Zel'dovich effect for clusters of galaxies.

2. The Sunyaev–Zel'dovich effect

Sunyaev and Zel'dovich [2–4] noted that the Compton scattering of the permeating 2.7 K cosmic microwave background photons by the high-temperature electrons of energy ∼10 keV in clusters of galaxies would cause a distortion in the Planck distribution of the 2.7 K photons. This effect has been confirmed and established by observation [5–16]. This effect makes possible the measurement of the Hubble constant for the expansion of the universe [5–16].

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Since the electrons in clusters of galaxies have rather high energy, of order 10 keV, a great need for a relativistic correction to the Sunyaev–Zel'dovich effect has arisen in recent years [17–21]. With regard to this point, Challinor and Lasenby [20] laid down a general foundation for the calculation of the relativistic corrections to the Sunyaev–Zel'dovich effect. The present authors [21] subsequently extended the work of Challinor and Lasenby. We furthermore carried out a direct numerical integration of the Boltzmann equation, thereby accurately examining the convergence of the power series expansion initiated by Challinor and Lasenby.

The Sunyaev–Zel'dovich effect is essentially a problem in the kinetic theory of gases. Since the elementary interaction is the well-known Compton scattering, the main problem in this research is that of how to calculate the higher-order terms. Fortunately, this can be done without too much difficulty. The results have very important consequences for astrophysics and cosmology.

3. The generalized Kompaneets equation

In this section we will extend the Kompaneets equation for the photon distribution function, taking into account the Compton scattering by electrons [22, 23], to the relativistic regime. We will formulate the kinetic equation for the photon distribution function using a relativistically covariant formalism [24, 25]. As a reference system, we choose the system which is fixed to the centre of mass of the cluster of galaxies. This choice of the reference system allows us to carry out all of the calculations in the most straightforward way. We will use the invariant amplitude for the Compton scattering as given by Berestetskii, Lifshitz and Pitaevskii [24] and by Buchler and Yueh [25].

The time evolution of the photon distribution function $n(\omega)$ is written as

$$\frac{\partial n(\omega)}{\partial t} = -2\int \frac{\mathrm{d}^3 p}{(2\pi)^3}\ \mathrm{d}^3 p'\ \mathrm{d}^3 k'\ W\left\{n(\omega)[1+n(\omega')]f(E) - n(\omega')[1+n(\omega)]f(E')\right\} \tag{3.1}$$

$$W = \frac{(e^2/4\pi)^2\overline{X}\delta^4(p+k-p'-k')}{2\omega\omega' E E'} \tag{3.2}$$

$$\overline{X} = -\left(\frac{\kappa}{\kappa'} + \frac{\kappa'}{\kappa}\right) + 4m^4\left(\frac{1}{\kappa} + \frac{1}{\kappa'}\right)^2 - 4m^2\left(\frac{1}{\kappa} + \frac{1}{\kappa'}\right) \tag{3.3}$$

$$\kappa = -2(pk) = -2\omega E\left(1 - \frac{|\boldsymbol{p}|}{E}\cos\alpha\right) \tag{3.4}$$

$$\kappa' = 2(pk') = 2\omega' E\left(1 - \frac{|\boldsymbol{p}|}{E}\cos\alpha'\right). \tag{3.5}$$

In the above, W is the transition probability corresponding to the Compton scattering. The four-momenta of the initial electron and photon are $p = (E, \boldsymbol{p})$ and $k = (\omega, \boldsymbol{k})$, respectively. The four-momenta of the final electron and photon are $p' = (E', \boldsymbol{p}')$ and $k' = (\omega', \boldsymbol{k}')$, respectively. The angles α and α' are the angles between $\boldsymbol{p}$ and $\boldsymbol{k}$, and between $\boldsymbol{p}$ and $\boldsymbol{k}'$, respectively. Throughout this paper, we use the natural $\hbar = c = 1$ units, unless otherwise stated explicitly.

By ignoring the degeneracy effects, we have the relativistic Maxwellian distribution for electrons with temperature T_e as follows:

$$f(E) = \left[\mathrm{e}^{\{(E-m)-(\mu-m)\}/k_B T_e} + 1\right]^{-1} \approx \mathrm{e}^{-\{K-(\mu-m)\}/k_B T_e} \tag{3.6}$$

where $K \equiv E - m$ is the kinetic energy of the initial electron, and $\mu - m$ is the non-relativistic chemical potential of the electron. We now introduce the quantities

$$x \equiv \frac{\omega}{k_B T_e} \tag{3.7}$$

$$\Delta x \equiv \frac{\omega' - \omega}{k_B T_e}. \tag{3.8}$$

Substituting equations (3.6)–(3.8) into equation (3.1), we obtain

$$\frac{\partial n(\omega)}{\partial t} = -2 \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, \mathrm{d}^3 p'\, \mathrm{d}^3 k'\, W f(E) \left\{[1 + n(\omega')]n(\omega) - [1 + n(\omega)]n(\omega')\mathrm{e}^{\Delta x}\right\}. \tag{3.9}$$

Equation (3.9) is our basic equation. One can numerically integrate this equation directly. We will perform this integration in section 4.

Following Challinor and Lasenby [20], we expand equation (3.9) in powers of Δx, assuming that $\Delta x \ll 1$. We obtain the Fokker–Planck expansion

$$\begin{aligned} \frac{\partial n(\omega)}{\partial t} = {} & 2\left[\frac{\partial n}{\partial x} + n(1+n)\right] I_1 + 2\left[\frac{\partial^2 n}{\partial x^2} + 2(1+n)\frac{\partial n}{\partial x} + n(1+n)\right] I_2 \\ & + 2\left[\frac{\partial^3 n}{\partial x^3} + 3(1+n)\frac{\partial^2 n}{\partial x^2} + 3(1+n)\frac{\partial n}{\partial x} + n(1+n)\right] I_3 \\ & + 2\left[\frac{\partial^4 n}{\partial x^4} + 4(1+n)\frac{\partial^3 n}{\partial x^3} + 6(1+n)\frac{\partial^2 n}{\partial x^2} + 4(1+n)\frac{\partial n}{\partial x} + n(1+n)\right] I_4 \\ & + \cdots \end{aligned} \tag{3.10}$$

where

$$I_k \equiv \frac{1}{k!} \int \frac{\mathrm{d}^3 p}{(2\pi)^3}\, \mathrm{d}^3 p'\, \mathrm{d}^3 k'\, W f(E)(\Delta x)^k. \tag{3.11}$$

Analytic integration of equation (3.11) is not possible except for by doing power series expansions of the integrand. Technically speaking, there are several choices for the expansion parameter of the integrand of equation (3.11). One could choose, for example, p/m, $K/m \equiv E/m - 1$ and $v \equiv p/E$. It is important to note that the analytic expressions for I_k obtained after the integration do not depend on the choice of the expansion parameter. It is also extremely important to note that the expansions in terms of these variables are *asymptotic expansions* in I_k. Therefore, not only is the convergence very slow but also the accuracy of the analytic expressions has to be carefully examined for the parameter region considered. This is one of our main subjects in the present paper.

Challinor and Lasenby [20] carried out a calculation up to $\mathrm{O}(\theta_e^3)$ terms, where $\theta_e = k_B T_e / mc^2$, T_e and m being the electron temperature and the electron mass, respectively. We will carry out a calculation up to $\mathrm{O}(\theta_e^5)$ terms in the present paper. The calculation of I_k has been performed with a symbolic manipulation computer algebra package *Mathematica*. We obtain

$$\begin{aligned} I_1 = {} & \frac{1}{2}\sigma_T N_e \theta_e x \Bigg\{ 4 - x + \theta_e\left(10 - \frac{47}{2}x + \frac{21}{5}x^2\right) + \theta_e^2\left(\frac{15}{2} - \frac{1023}{8}x + \frac{567}{5}x^2 - \frac{147}{10}x^3\right) \\ & + \theta_e^3\left(-\frac{15}{2} - \frac{2505}{8}x + \frac{9891}{10}x^2 - \frac{9551}{20}x^3 + \frac{1616}{35}x^4\right) \\ & + \theta_e^4\left(\frac{135}{32} - \frac{30\,375}{128}x + \frac{177\,849}{40}x^2 - \frac{472\,349}{80}x^3 + \frac{63\,456}{35}x^4 - \frac{940}{7}x^5\right)\Bigg\} \end{aligned} \tag{3.12}$$

$$I_2 = \frac{1}{2}\sigma_T N_e\theta_e x^2\left\{1 + \theta_e\left(\frac{47}{2} - \frac{63}{5}x + \frac{7}{10}x^2\right) + \theta_e^2\left(\frac{1023}{8} - \frac{1302}{5}x + \frac{161}{2}x^2 - \frac{22}{5}x^3\right)\right.$$
$$+ \theta_e^3\left(\frac{2505}{8} - \frac{10\,647}{5}x + \frac{38\,057}{20}x^2 - \frac{2829}{7}x^3 + \frac{682}{35}x^4\right)$$
$$+ \theta_e^4\left(\frac{30\,375}{128} - \frac{187\,173}{20}x + \frac{17\,01\,803}{80}x^2 - \frac{44\,769}{4}x^3\right.$$
$$\left.\left. + \frac{61\,512}{35}x^4 - \frac{510}{7}x^5\right)\right\} \tag{3.13}$$

$$I_3 = \frac{1}{2}\sigma_T N_e\theta_e x^3\left\{\theta_e\left(\frac{42}{5} - \frac{7}{5}x\right) + \theta_e^2\left(\frac{868}{5} - \frac{658}{5}x + \frac{88}{5}x^2 - \frac{11}{30}x^3\right)\right.$$
$$+ \theta_e^3\left(\frac{7098}{5} - \frac{14\,253}{5}x + \frac{8084}{7}x^2 - \frac{3503}{28}x^3 + \frac{64}{21}x^4\right)$$
$$+ \theta_e^4\left(\frac{62\,391}{10} - \frac{614\,727}{20}x + 28\,193x^2 - \frac{123\,083}{16}x^3\right.$$
$$\left.\left. + \frac{14\,404}{21}x^4 - \frac{344}{21}x^5\right)\right\} \tag{3.14}$$

$$I_4 = \frac{1}{2}\sigma_T N_e\theta_e x^4\left\{\frac{7}{10}\theta_e + \theta_e^2\left(\frac{329}{5} - 22x + \frac{11}{10}x^2\right)\right.$$
$$+ \theta_e^3\left(\frac{14\,253}{10} - \frac{9297}{7}x + \frac{7781}{28}x^2 - \frac{320}{21}x^3 + \frac{16}{105}x^4\right)$$
$$+ \theta_e^4\left(\frac{614\,727}{40} - \frac{124\,389}{4}x + \frac{239\,393}{16}x^2 - \frac{7010}{3}x^3\right.$$
$$\left.\left. + \frac{12\,676}{105}x^4 - \frac{11}{7}x^5\right)\right\} \tag{3.15}$$

$$I_5 = \frac{1}{2}\sigma_T N_e\theta_e x^5\left\{\theta_e^2\left(\frac{44}{5} - \frac{11}{10}x\right) + \theta_e^3\left(\frac{18\,594}{35} - \frac{36\,177}{140}x + \frac{192}{7}x^2 - \frac{64}{105}x^3\right)\right.$$
$$\left. + \theta_e^4\left(\frac{124\,389}{10} - \frac{10\,67\,109}{80}x + 3696x^2 - \frac{5032}{15}x^3 + \frac{66}{7}x^4 - \frac{11}{210}x^5\right)\right\} \tag{3.16}$$

$$I_6 = \frac{1}{2}\sigma_T N_e\theta_e x^6\left\{\frac{11}{30}\theta_e^2 + \theta_e^3\left(\frac{12\,059}{140} - \frac{64}{3}x + \frac{32}{35}x^2\right)\right.$$
$$\left. + \theta_e^4\left(\frac{355\,703}{80} - \frac{8284}{3}x + \frac{6688}{15}x^2 - 22x^3 + \frac{11}{42}x^4\right)\right\} \tag{3.17}$$

$$I_7 = \frac{1}{2}\sigma_T N_e\theta_e x^7\left\{\theta_e^3\left(\frac{128}{21} - \frac{64}{105}x\right) + \theta_e^4\left(\frac{16\,568}{21} - \frac{30\,064}{105}x + \frac{176}{7}x^2 - \frac{11}{21}x^3\right)\right\} \tag{3.18}$$

$$I_8 = \frac{1}{2}\sigma_T N_e\theta_e x^8\left\{\frac{16}{105}\theta_e^3 + \theta_e^4\left(\frac{7516}{105} - \frac{99}{7}x + \frac{11}{21}x^2\right)\right\} \tag{3.19}$$

$$I_9 = \frac{1}{2}\sigma_T N_e\theta_e x^9\left\{\theta_e^4\left(\frac{22}{7} - \frac{11}{42}x\right)\right\} \tag{3.20}$$

$$I_{10} = \frac{1}{2}\sigma_T N_e \theta_e x^{10} \left\{ \frac{11}{210}\theta_e^4 \right\} \tag{3.21}$$

where σ_T is the Thomson scattering cross section and N_e is the electron number density. The expansion parameter θ_e is defined by

$$\theta_e \equiv \frac{k_B T_e}{mc^2}. \tag{3.22}$$

In deriving equations (3.12)–(3.21), we have ignored $O(\theta_e^6)$ contributions. Using equations (3.12)–(3.21), one can show that the photon number is conserved order by order in terms of the expansion parameter θ_e.

We now apply the present result for the generalized Kompaneets equation to the Sunyaev–Zel'dovich effect for clusters of galaxies. We assume the initial photon distribution of the CMB radiation to be Planckian with temperature T_0:

$$n_0(X) = \frac{1}{e^X - 1} \tag{3.23}$$

where

$$X \equiv \frac{\omega}{k_B T_0}. \tag{3.24}$$

Substituting equation (3.23) and equations (3.12)–(3.21) into equation (3.10), and assuming $T_0/T_e \ll 1$, one obtains the following expression for the fractional distortion of the photon spectrum:

$$\frac{\Delta n(X)}{n_0(X)} = \frac{y\theta_e X e^X}{e^X - 1}\left[Y_0 + \theta_e Y_1 + \theta_e^2 Y_2 + \theta_e^3 Y_3 + \theta_e^4 Y_4\right] \tag{3.25}$$

$$Y_0 = -4 + \tilde{X} \tag{3.26}$$

$$Y_1 = -10 + \frac{47}{2}\tilde{X} - \frac{42}{5}\tilde{X}^2 + \frac{7}{10}\tilde{X}^3 + \tilde{S}^2\left(-\frac{21}{5} + \frac{7}{5}\tilde{X}\right) \tag{3.27}$$

$$\begin{aligned} Y_2 = {} & -\frac{15}{2} + \frac{1023}{8}\tilde{X} - \frac{868}{5}\tilde{X}^2 + \frac{329}{5}\tilde{X}^3 - \frac{44}{5}\tilde{X}^4 + \frac{11}{30}\tilde{X}^5 \\ & + \tilde{S}^2\left(-\frac{434}{5} + \frac{658}{5}\tilde{X} - \frac{242}{5}\tilde{X}^2 + \frac{143}{30}\tilde{X}^3\right) + \tilde{S}^4\left(-\frac{44}{5} + \frac{187}{60}\tilde{X}\right) \end{aligned} \tag{3.28}$$

$$\begin{aligned} Y_3 = {} & \frac{15}{2} + \frac{2505}{8}\tilde{X} - \frac{7098}{5}\tilde{X}^2 + \frac{14\,253}{10}\tilde{X}^3 - \frac{18\,594}{35}\tilde{X}^4 + \frac{12\,059}{140}\tilde{X}^5 - \frac{128}{21}\tilde{X}^6 + \frac{16}{105}\tilde{X}^7 \\ & + \tilde{S}^2\left(-\frac{7098}{10} + \frac{14\,253}{5}\tilde{X} - \frac{102\,267}{35}\tilde{X}^2 + \frac{156\,767}{140}\tilde{X}^3 \right. \\ & \left. - \frac{1216}{7}\tilde{X}^4 + \frac{64}{7}\tilde{X}^5\right) \\ & + \tilde{S}^4\left(-\frac{18\,594}{35} + \frac{205\,003}{280}\tilde{X} - \frac{1920}{7}\tilde{X}^2 + \frac{1024}{35}\tilde{X}^3\right) \\ & + \tilde{S}^6\left(-\frac{544}{21} + \frac{992}{105}\tilde{X}\right) \end{aligned} \tag{3.29}$$

$$\begin{aligned} Y_4 = {} & -\frac{135}{32} + \frac{30\,375}{128}\tilde{X} - \frac{62\,391}{10}\tilde{X}^2 + \frac{614\,727}{40}\tilde{X}^3 - \frac{124\,389}{10}\tilde{X}^4 \\ & + \frac{355\,703}{80}\tilde{X}^5 - \frac{16\,568}{21}\tilde{X}^6 + \frac{7516}{105}\tilde{X}^7 - \frac{22}{7}\tilde{X}^8 + \frac{11}{210}\tilde{X}^9 \end{aligned}$$

$$+ \tilde{S}^2\left(-\frac{62\,391}{20} + \frac{614\,727}{20}\tilde{X} - \frac{1368\,279}{20}\tilde{X}^2 + \frac{4624\,139}{80}\tilde{X}^3 - \frac{157\,396}{7}\tilde{X}^4 + \frac{30\,064}{7}\tilde{X}^5 - \frac{2717}{7}\tilde{X}^6 + \frac{2761}{210}\tilde{X}^7\right)$$
$$+ \tilde{S}^4\left(-\frac{124\,389}{10} + \frac{6046\,951}{160}\tilde{X} - \frac{248\,520}{7}\tilde{X}^2 + \frac{481\,024}{35}\tilde{X}^3 - \frac{15\,972}{7}\tilde{X}^4 + \frac{18\,689}{140}\tilde{X}^5\right)$$
$$+ \tilde{S}^6\left(-\frac{70\,414}{21} + \frac{465\,992}{105}\tilde{X} - \frac{11\,792}{7}\tilde{X}^2 + \frac{19\,778}{105}\tilde{X}^3\right)$$
$$+ \tilde{S}^8\left(-\frac{682}{7} + \frac{7601}{210}\tilde{X}\right) \quad (3.30)$$

where

$$y \equiv \sigma_T \int \mathrm{d}\ell\ N_e \quad (3.31)$$

$$\tilde{X} \equiv X \coth\left(\frac{X}{2}\right) \quad (3.32)$$

$$\tilde{S} \equiv \frac{X}{\sinh(X/2)}. \quad (3.33)$$

Note that the analytic forms of Y_0, Y_1 and Y_2 in equations (3.26)–(3.28) agree with the results obtained by Challinor and Lasenby [20]. Finally we define the distortion of the spectral intensity as follows:

$$\Delta I = \frac{X^3}{\mathrm{e}^X - 1}\frac{\Delta n(X)}{n_0(X)}. \quad (3.34)$$

4. Analysis of the convergence of the power series

We now carefully study the convergence of the analytic expressions of equations (3.25) and (3.34). In order to perform this task, first of all, we integrate equation (3.9) directly by numerical integration. We confirm that the total photon number is conserved with excellent accuracy ($<10^{-9}$) in the numerical integration. We are now ready to compare the present numerical results with those obtained from the analytic expressions of equations (3.25) and (3.34) for various X–T_e regions and to investigate the accuracy of the analytic expressions.

4.1. The Rayleigh–Jeans region

In the Rayleigh–Jeans limit where $X \to 0$, equation (3.25) is further simplified:

$$\frac{\Delta n(X)}{n_0(X)} \longrightarrow -2y\theta_e\left[1 - \frac{17}{10}\theta_e + \frac{123}{40}\theta_e^2 - \frac{1989}{280}\theta_e^3 + \frac{14\,403}{640}\theta_e^4\right]. \quad (4.1)$$

As is seen explicitly from equation (4.1), the convergence of the power expansion is very fast in the $X \to 0$ limit. Furthermore, we show in figure 1 the T_e-dependence of the spectral intensity distortion equation (3.34) for $X = 1$. As is expected, the convergence is extremely fast for $k_B T_e \leqslant 50$ keV. Relativistic corrections from higher-than-O(θ_e^3) terms are almost negligible in this region. So far the Sunyaev–Zel'dovich effects have only been measured

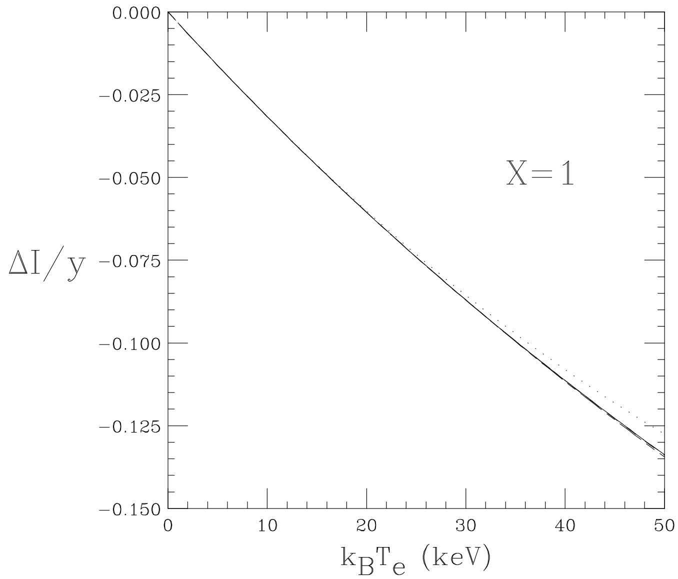

Figure 1. The spectral intensity distortion $\Delta I/y$ as a function of k_BT_e for $X = 1$. The solid curve shows the result of the numerical integration. The dotted curve shows the contribution of the first two terms in equation (3.25). The long–short dashed curve shows the contribution of the first three terms. The chain curve shows the contribution from the first four terms. (These latter two curves cannot be distinguished in this figure.) Finally the dashed curve shows the contribution from all of the terms in equation (3.25).

in the Rayleigh–Jeans region. Therefore one can reliably apply the analytic expressions of equations (3.25), (3.34) and (4.1) to the analysis of the observed data.

In passing, we remark that the form of equation (4.1) is meaningful only in an idealized situation. In order for the higher-order terms to have a physical meaning, it is necessary that the electron distribution is rigorously given by the relativistic Maxwellian distribution equation (3.6) with a precisely determined temperature T_e. In a real observation, the electron temperature T_e has a significant amount of observational error. This thereby restricts the precision of formula (4.1).

4.2. *The* $X \approx 4$ *region*

As one can see from equation (3.26), the leading-order contribution Y_0 vanishes at $\tilde{X} = 4 \approx X$. Therefore, higher-order corrections become more important in this region. In figure 2 we have plotted the T_e-dependence of the fractional spectral distortion at $X = 4$. It is seen that the chain line is closest to the exact result. It should be emphasized here that the chain line is the result including only the first four terms in equation (3.25). The dashed curve, which includes all of the terms in equation (3.25), shows a poorer agreement with the exact result. This means that the power series expansion in θ_e in this region is not convergent but asymptotic for large T_e. We conclude that the analytic expression which includes up to $\mathrm{O}(\theta_e^5)$ terms is reliable for $k_BT_e \leqslant 15$ keV in the $X \approx 4$ region. We recommend that the analytic expression which includes up to $\theta_e^3 Y_3$ terms (the chain curve) be used for the analysis of the observational data for 15 keV $< k_BT_e <$ 30 keV, $X \approx 4$.

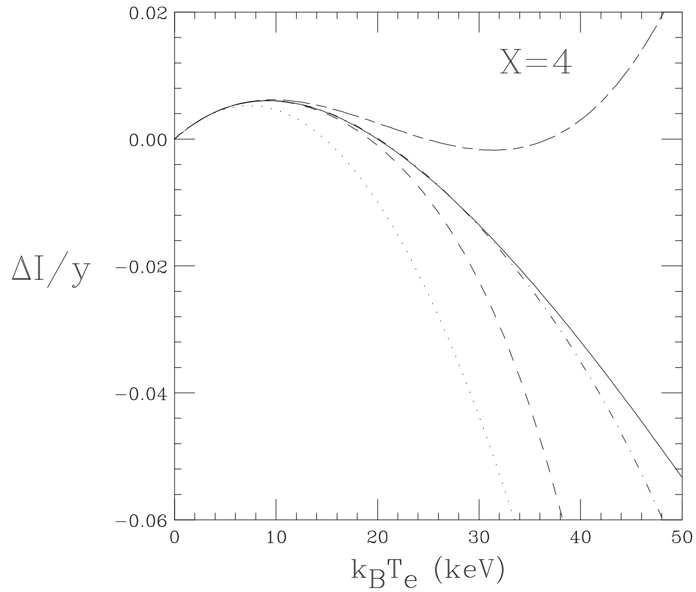

Figure 2. As figure 1, but for $X = 4$.

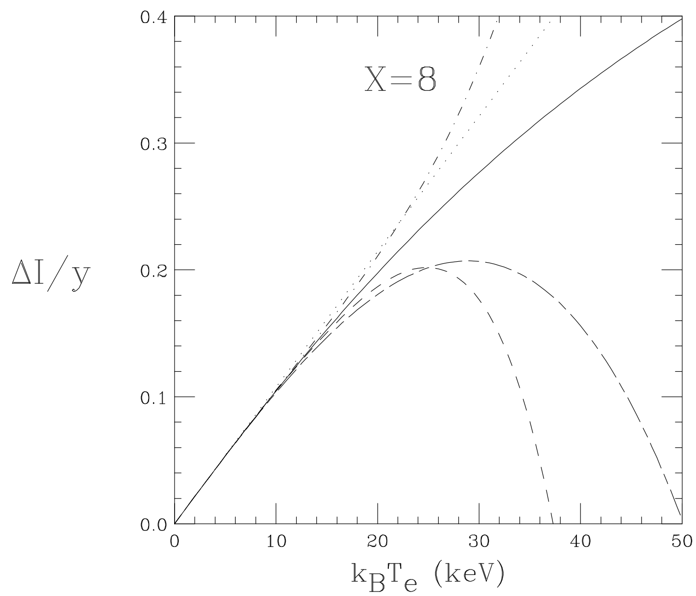

Figure 3. As figure 1, but for $X = 8$.

4.3. The Wien region

We now study the Wien region where $X > 4$. As was mentioned earlier, the Sunyaev–Zel'dovich effects have been so far studied observationally only in the $X \ll 1$ region. However, the effects will be observed in the Wien region in the future. For illustrative purposes, we show the T_e-dependence of the spectral intensity distortion of equation (3.34)

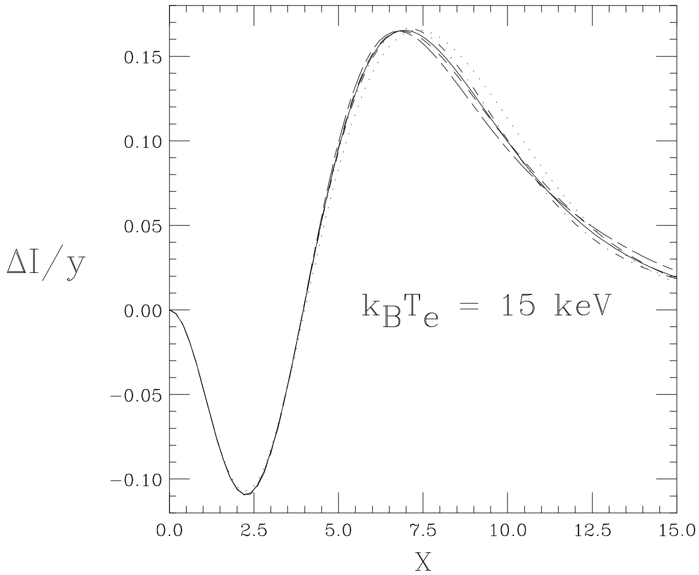

Figure 4. The spectral intensity distortion $\Delta I/y$ as a function of X for $k_BT_e = 15$ keV. See figure 1 for the key to the curves.

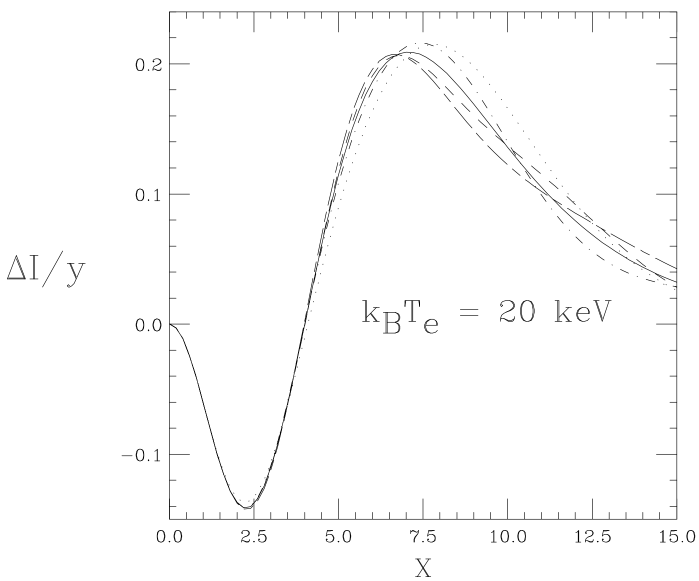

Figure 5. As figure 4, but for $k_BT_e = 20$ keV.

at $X = 8$ in figure 3. The convergence is very slow. All of the curves are diverging quickly from the solid curve (the exact result) for $k_BT_e > 30$ keV. We conclude that the analytic expression including up to $O(\theta_e^5)$ terms is reliable for $k_BT_e \leqslant 15$ keV. In figures 4 and 5 we show $\Delta I/y$ for $k_BT_e = 15$ keV and $k_BT_e = 20$ keV, respectively. We confirm the good accuracy of the analytic expression for $k_BT_e = 15$ keV.

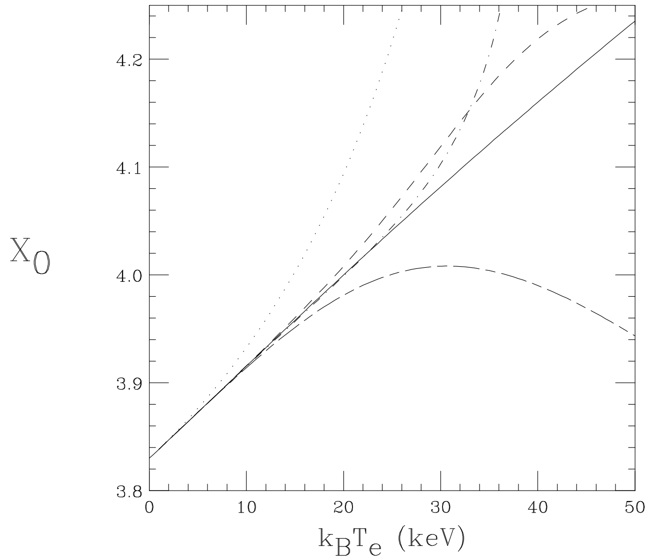

Figure 6. The crossover frequency X_0 as a function of k_BT_e. The solid curve shows the result of the numerical integration. The dotted curve shows the contribution of the first two terms in equation (3.25). The long–short dashed curve shows the contribution of the first three terms. The chain curve shows the contribution from the first four terms. Finally, the dashed curve shows the contribution from all of the terms in equation (3.25).

4.4. The crossover frequency

Finally we study the crossover frequency X_0, at which the spectral intensity distortion vanishes. It is known that the accurate determination of the X_0-values is extremely important for the study of the Sunyaev–Zel'dovich effects [18]. In figure 6, we have plotted the T_e-dependence of X_0 for $k_BT_e \leqslant 50$ keV calculated from the analytic expressions and also by numerical integration (solid curve). The numerical result is well approximated as a linear function of θ_e for $k_BT_e < 20$ keV. It starts to deviate from the linear form for $k_BT_e > 20$ keV. We have fitted the numerical result as follows:

$$X_0 \approx 3.830(1 + 1.1674\theta_e - 0.8533\theta_e^2). \tag{4.2}$$

The errors of this fitting function are less than 1×10^{-3} for $0 \leqslant k_BT_e \leqslant 50$ keV.

Rephaeli and Yankovitch [18] discuss the consequence of the relativistic shift of the crossover frequency for the value of the peculiar velocity of the cluster measured with the use of the kinematic Sunyaev–Zel'dovich effect. For a cluster with $k_BT_e = 13.8$ keV, one obtains an error of 20 km s^{-1} in the value of the peculiar velocity deduced for the SUZIE experiment corresponding to an accuracy of 1×10^{-3} in equation (4.2), by interpolating Rephaeli and Yankovitch's estimation. Therefore, one concludes that the current level of the observational accuracy has not reached the accuracy provided by the theoretical formula (4.2).

Another implication of the accuracy, 1×10^{-3}, of equation (4.2) is the following. Let us consider a cluster with the plasma temperature $k_BT_e = 10$ keV making a proper motion with the velocity $v = 1000$ km s^{-1}. Then the effect of this proper motion on the Sunyaev–Zel'dovich effect will be of order $(v/c)^2/(k_BT_e/mc^2) = 5 \times 10^{-4}$. Thus the kinematic effect of the proper motion of the cluster will not exceed the accuracy of equation (4.2).

5. Concluding remarks

We have reviewed the recent progress in the study of the relativistic correction to the Sunyaev–Zel'dovich effect for clusters of galaxies in detail. This is a very good example of a subject in the field of the kinetic theory of gases. (As we read from the original paper by Kompaneets [22], L D Landau was interested in the problem of the Compton scattering of photons by electrons.) It deals with a system of very simple elementary particles (photons and electrons) and a very simple interaction between them (Compton scattering). Yet it produces very rich physical effects known as the Sunyaev–Zel'dovich effect. It is extremely interesting to note that this situation is realized in clusters of galaxies and that the effect has been confirmed and established by observation. The Sunyaev–Zel'dovich effect also enables one to measure the Hubble constant for the cosmic expansion. This is again a good example of bridging from microscopic physics (Compton scattering of photons by electrons) to macroscopic physics (astrophysics and cosmology). In order to achieve this goal, one makes full use of the kinetic theory of gases. We hope that we have introduced to condensed matter physicists how the universe provides us with fascinating new physical phases under extreme conditions.

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